The LFX Canadian Model

Version 1.0 - Technical Documentation

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Introduction
LFXCM is a hybrid Input Output/Computable General Equilibrium (IO/CGE) model of the Canadian economy which resolves private sector activity into 26 sectors with associated outputs in each of ten provinces plus the far north territories. Within each province it identifies the following industry sectors:

1 Agriculture Fishing and Trapping
2 Forestry and Logging
3 Oil Sands
4 Conventional Crude Oil
5 Natural Gas
6 Oil and Gas Support Activities
7 Coal
8 Other Mining
9 Electricity
10 Other Utilities incl Gas Distribution
11 Construction
12 Food Production
13 Semi-durables
14 Refined Fuels
15 Other Petrochemicals
16 Cement and Concrete
17 Automotive Parts and Assembly
18 Other Manufacturing
19 Wholesale and Retail Sales
20 Air Rail & Bus Transportation
21 Gas Pipelines
22 Crude Pipelines
23 Trucking Courier and Storage
24 Media, Banking, Finance, Information and related Professional Services
25 Education and Health
26 Entertainment, Travel, Restaurants and Miscellaneous Services.

The list of commodities is the same and all outputs are assigned to the corresponding sector. Petroleum products are distinguished between fuels and those used for non-combustion applications. The model resolves output, capital demand, labour demand and intermediate input demand for every commodity in every sector for each province, calibrated so as to reproduce the 2016 provincial-level Canadian input-output tables.

Final demand categories include Households, Government, Gross Fixed Capital Formation (GFCF), Domestic (inter-provincial) Exports and Foreign Exports. Output includes net supply by domestic sectors, Domestic Imports and International Imports. Overall GFCF is determined by the profitability in a sector compared to base case profitability, with IO shares allocated using the 2016 calibration. Household savings are determined in this version of the model as a fixed fraction of income. GFCF is funded by available savings in the economy plus foreign borrowing.

Nesting structure

Households and firms are represented using nested CES share functions. The household nest sequence is as follows:

<table>
<tr><td>Savings</td><td></td><td></td><td></td></tr>
<tr><td>Leisure</td><td></td><td></td><td></td></tr>
<tr><td rowspan="7">Consumption</td><td rowspan="3">ENERGY & TRANSPORT</td><td>UTILITIES</td><td>Electricity
Other Utilities
Gas Pipeline Services</td></tr>
<tr><td>FUELS</td><td>Natural Gas
Coal
Gasoline
Petrochemicals</td></tr>
<tr><td>TRANSPORT</td><td>Oil Pipeline Services
Air, Rail & Bus
Trucking & Storage</td></tr>
<tr><td rowspan="2">GOODS</td><td>BASIC GOODS</td><td>Conventional Crude
Oil Sands
Agriculture
Forest Products
Mining</td></tr>
<tr><td>PRODUCED GOODS</td><td>Cement
Semi-durables
Auto Parts & Assembly
Other Manufacturing
Food</td></tr>
<tr><td rowspan="2">SERVICES</td><td>PROFESSIONAL SERVICES</td><td>Entertainment
Construction
Media, Finance Etc
Sales & Retail</td></tr>
<tr><td>OTHER SERVICES</td><td>Oil & Gas Support
Education & Health</td></tr>
</table>

The nesting structure for firms is essentially the same except the top level combines intermediate inputs with labour and capital demand to yield output.

LFXCM can accommodate a unique elasticity value for each nest for each province. Initial values have been selected based on literature search and trial-and-error, but are subject to adjustment as more information is acquired. CES function scaling parameters are calibrated to reproduce budget shares based on the 2016 StatsCan provincial IO tables.

All program components and functions are written in R.

Calibration

The LFXCM program assimilates the entire 2016 Statistics Canada province-level Input-Output table which resolves about 500 sectors and 500 commodities. These are aggregated into the 26 categories listed above. The condensed tables are then used to calibrate all share parameters and tax parameters.

Factors of Production

Factors of production include employment (by sector and province) and capital. Capital stock valuations by sector and province are developed as scalar multiples of the operating surplus reported in the input-output tables, averaged over 2014-2016. The model also generates real and nominal capital demand in each solution, yielding an endogenous capital utilization rate.

Tax Detail

Separate intermediate tax rates by industry and province are computed using the CANSIM Input-Output table values of output and input taxes net of subsidies on outputs and inputs, with the current federal carbon charge ($20/tonne) added in the policy base case. Households also pay consumption taxes computed at the provincial level to take into account PST and HST rates across the province as well as the federal carbon tax levy. Households also pay income taxes which are computed using the national total income tax revenues as recorded by Statistics Canada. The same average income tax rate applies equally to labour and capital income.

Share Functions

These functional forms are drawn from Shoven and Whalley *Applying General Equilibrium* and Berck and Sydsaeter *Economists' Mathematical Manual*.

Given a set of intermediate input prices the model determines input-output coefficients for each sector in each province. The input-output coefficients vary as relative prices change. The IO coefficients begin with the assumption that, within a nest consisting of (for example) two inputs (x_1, x_2) the firm chooses them to maximize

$$y = \left((a_1 x_1)^{\frac{\sigma-1}{\sigma}} + (a_2 x_2)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \text{ subject to } p_1 x_1 + p_2 x_2 = C$$

where $\sigma =$ the elasticity of substitution. Note $a_i = w_i^{\frac{1}{\sigma-1}}$ where w_i are the base case real shares (= nominal shares assuming base case prices = 1).

The input-output coefficients consistent with the optimal solution are

$$\frac{x_i}{y} = p_i^{-\sigma} a_i^{\sigma-1} \left(\left(\frac{p_1}{a_1}\right)^{1-\sigma} + \left(\frac{p_2}{a_2}\right)^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}}$$

The zero-profit condition implies $p_y y = p_1 x_1 + p_2 x_2$ therefore the nest price is

$$p_y = p_1 \frac{x_1}{y} + p_2 \frac{x_2}{y}$$

The household model uses nominal shares. Given prices and total income, the consumer maximizes a utility function. Standard CES forms are:

Shoven-Whalley form: $U = \left(\Sigma_i \, \alpha_{\mathrm{i}}^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$ where α_i are budget shares.

Berck-Sydsaeter form: $U = \left(\Sigma_i a_i^{\frac{\sigma-1}{\sigma}} \, x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$ where $a_i = \alpha_i^{\frac{1}{\sigma-1}}$ and α_i are budget shares

Note $a_i^{\frac{\sigma-1}{\sigma}} = \alpha_i^{\frac{1}{\sigma}} \Rightarrow a_i = \alpha_i^{\frac{1}{\sigma-1}} \Rightarrow a_i^{\sigma-1} = \alpha_i$.

The optimal nominal shares according to Shoven-Whalley are:

$$x_i = \frac{\alpha_i I}{p_i^{\sigma} \Sigma_j \alpha_j p_j^{1-\sigma}}$$

$$\Rightarrow \frac{p_i x_i}{I} = \frac{\alpha_i p_i}{p_i^{\sigma} \Sigma_j \alpha_j p_j^{1-\sigma}} = \frac{\alpha_i p_i^{1-\sigma}}{\Sigma_j \alpha_j p_j^{1-\sigma}}$$

$$= \frac{a_i^{-(1-\sigma)} p_i^{1-\sigma}}{\Sigma_j a_i^{-(1-\sigma)} p_j^{1-\sigma}}$$

$$= \frac{\left(\frac{p_i}{a_i}\right)^{1-\sigma}}{\Sigma_j \left(\frac{p_j}{a_j}\right)^{1-\sigma}}$$

Consumer Model – Top Level

The utility function combines demand for leisure H and consumption C with associated prices w and p, time endowment T (which equals leisure H plus labour L) and exogenous income Y. The utility function is

$$U = \frac{\gamma}{\alpha} H^{\alpha} + C$$

where γ is a scaling parameter. This is optimized against the budget constraint $wH + pC = Tw + Y$ using a Lagrangian function

$$\ell = U - \lambda(wH + pC - Tw - Y)$$

The first-order conditions are:

$$\ell_C = 1 - \lambda p = 0 \Rightarrow \lambda = \frac{1}{p}$$

$$\ell_H = \gamma H^{\alpha-1} - \lambda w = 0 \Rightarrow H = \left(\frac{1}{\gamma}\right)^{\frac{1}{\alpha-1}} \left(\frac{w}{p}\right)^{\frac{1}{\alpha-1}}$$

These can be solved to yield a labour supply function

$$L = T - \theta \left(\frac{w}{p}\right)^{\sigma}$$

where $\theta = \gamma^{\frac{1}{1-\alpha}}$ is a scaling parameter and $\sigma = \frac{1}{\alpha-1}$ is the elasticity of leisure demand with respect to the real wage rate. Values from -0.3 to -0.7 are typically used and results are examined for sensitivity to this parameter choice.

Regulatory Rents and Policy Experiments

The cost of certain regulations is akin to a tax-induced "Harberger triangle" or deadweight loss, except that the revenue portion does not accrue to the government instead it is dissipated and is unavailable to the economy. For example, suppose a regulation is introduced requiring construction firms to change procedures in such a way that the cost of building a home rises by 20 percent, but at the end of the process, the extra cost does not yield a 20 percent bigger house, instead the same size house has been created. In this case the production cost is scaled up by 20 percent but the increased selling price does not accrue as revenue to the home builder, instead it is offset by decreased productivity of the inputs. The LFXCM builds a number of such regulatory inefficiencies into the base case of the model, including in the electricity and refining sectors, based on relative changes over time among provinces in the marginal cost of producing equivalent outputs. The LFXCM then tracks the national costs of compliance with these regulations. No attempt is made within the LFXCM to quantify the intended benefits associated with these regulations, although such estimates can be made using the model outputs.

Policy experiments can be run in the LFXCM in which a new policy is represented in the form of changes to the pre-existing regulatory constraint structure, changes to factor supplies, changes in any of the provincial or federal tax and subsidy rates, and so forth. Numerous metrics are available for determining the costs and benefits of the policy, including provincial utility, real GDP, real consumption, employment, changes in the equity value of the capital stock, etc.

Model Solution

Initial interprovincial export and import levels are taken from the 2016 provincial IO tables which balance to zero at the national level. They are rescaled endogenously based on income levels which may change during a policy experiment.

International exports and imports are taken from the 2016 provincial IO tables. The exchange rate is held fixed in this version of the model. Export demands are responsive to sectoral marginal costs of production, such that an increase in domestic production costs lead to a decline in real exports.

Government revenue is determined endogenously based on tax rates as described above. Transfers to households and labour demand are fixed at 2016 levels in the policy base case, and government purchases of goods and services are based on 2016 levels rescaled to match growth or decline in the labour market. The government budget surplus or deficit is thus endogenous.

GFCF is driven by a demand equation in which 2016 investments by sector and province are scaled up or down based on current rates of return to investment in a sector. The rate of return is determined using the endogenous capital demand less regulatory rents (explained below) relative to the base case implicit capital stock. The nominal level of investment determines provincial investment needs. The funds available for investment by province are determined as the sum of household savings, the government surplus and the capital account surplus less all taxes on traded goods. The capital account surplus is the negative of the current account balance. If funds needed for investment exceed those available the balance is made up by foreign borrowing, which is counted against a province's overall capital equity.

Within a province, given prices, tax rates, government spending and trade parameters the model yields an intermediate IO coefficient matrix A, and final demands for consumption C, government purchases G, investment or Gross Fixed Capital Formation I, exports X and imports M. Denote $C + I + G + X - M = F$. If real output is Q the Leontief condition is $AQ + F = Q$.The model solves for output using the matrix equation

$$Q = (I - A)^{-1}F.$$

Then input-output coefficients for labour and capital are used to determine labour and capital demands by sector and province. This implicitly uses a constant returns to scale assumption to clear all goods and service markets. Exogenous restrictions are imposed on the Education and Health care sector in some provinces to limit its expansion since it is primarily governed by government policy and cannot respond freely to market conditions.

Since the Leontief equation is solved for each province, and some provinces are net importers of some goods (for example, Ontario imports crude oil for refining), the equilibrium output level can be negative. If the labour IO coefficient were applied it would yield a negative demand for labour. This is also an implication of the "cross-hauling" phenomenon in which provinces can both import and export the same commodity, such as food for instance. The relevant labour demand level is therefore based on final demand before subtracting imports, which equals $(C + I + G + X)$. This yields, for example, an employment level of zero for oil sands production in Ontario, which is the appropriate estimate. The model uses the pre-import final demand amount as the basis for analyzing labour demand changes in each sector and province.

The model adjusts the national wage rate to clear the national labour market, and the capital market also clears as a consequence of budget constraints. The provincial labour markets do not necessarily clear: there can be surpluses or shortage of labour within a province but they add up to zero nationally. The program verifies that Walras' Law holds at every iteration.

Policy Parameters

Separate tax rates for each commodity in each province are tracked, as are labour, capital and carbon taxes. Regulations are modeled as exogenous shifts to input costs or sectoral supply curves. Regulatory details can be specified down to the sectoral level within each province. A regulatory measure is quantified as a scarcity rent as described above.

Greenhouse gas emissions are computed using coefficients calibrated on consumption of coal, natural gas, refined fuels and cement production so as to reproduce the 2016 national carbon dioxide emissions inventory.

The costs and benefits of policy changes can be computed in numerous ways depending on the needs of the application, including changes in indirect utility, equivalent variations, real GDP, household consumption, employment, marginal regulatory rents, and so forth.

Model Calibration of Greenhouse Gases

Sources:
Marland, G., and R.M. Rotty. 1984. Carbon dioxide emissions from fossil fuels: A procedure for estimation and results for 1950-82. *Tellus* 36(B):232-61.

BP Annual Statistical Review of Energy https://www.bp.com/content/dam/bp/business-sites/en/global/corporate/pdfs/energy-economics/statistical-review/bp-stats-review-2020-full-report.pdf

NRCan Energy FactBook

Emission coefficients for coal, petroleum liquids and natural gas are derived as follows. BP reports that in 2016 Canada consumed 2,503 thousand barrels per day of oil, 3.82 exajoules of natural gas and 0.78 exajoules of coal. Marland and Rotty estimate carbon emission coefficients for natural gas as 13.7 tC /TJ; for oil 0.85 tC/tonne oil and for coal 0.75 tC/tonne coal.

For oil, 620.5 million barrels of oil annually at 0.136 tonnes per barrel implies 84.4 Mt oil and 71.7 MtC. Using a conversion factor of 11/3 implies 263.0MtCO2.

For natural gas 3,820,000 TJ implies 52.3 Mt Carbon and using a conversion factor of 11/3 this implies 191.9 MtCO2.

For coal 780,000 TJ converts to mass using 29.31×10^9 J/t (Marland and Rotty) yielding 26.6 Mt coal, 20.0 MtC and 73.2 Mt CO2.

Canada's IPCC Emission Inventory (https://unfccc.int/documents/65715) lists 6Mt CO2 emissions associated with cement production.