



# DETAILED DESCRIPTION OF THE LFX-CM5 MODEL

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## 1 INTRODUCTION

LFXCM Version 5.0 is a dynamic hybrid Input Output/Computable General Equilibrium (IO/CGE) model of the Canadian economy which resolves annual private sector activity into 26 sectors with associated outputs in each of ten provinces plus the far north territories. Within each province it identifies inputs and outputs for the following sectors:

- 1 Agriculture Fishing and Trapping
- 2 Forestry and Logging
- 3 Oil Sands
- 4 Conventional Crude Oil
- 5 Natural Gas
- 6 Oil and Gas Support Activities
- 7 Coal
- 8 Other Mining
- 9 Electricity
- 10 Other Utilities incl Gas Distribution
- 11 Construction
- 12 Food Production
- 13 Semi-durables
- 14 Refined Fuels
- 15 Other Petrochemicals
- 16 Cement and Concrete
- 17 Automotive Parts and Assembly
- 18 Other Manufacturing
- 19 Wholesale and Retail Sales
- 20 Air Rail & Bus Transportation
- 21 Gas Pipelines
- 22 Crude Pipelines
- 23 Trucking Courier and Storage
- 24 Media, Banking, Finance, Information and related Professional Services
- 25 Education and Health
- 26 Entertainment, Travel, Restaurants and Miscellaneous Services.

The list of commodities is the same and all outputs are assigned to the corresponding sector. Petroleum products are distinguished between fuels and those used for non-combustion applications. The model resolves output, capital demand, labour demand and intermediate input demand for every commodity in every sector for each province, calibrated so as to reproduce the 2019 provincial-level Canadian input-output tables.

Final demand categories include Households, Government, Gross Fixed Capital Formation (GFCF), Domestic (inter-provincial) Exports and Foreign Exports. Output includes net supply by domestic sectors, Domestic Imports and International Imports.

The prices of labour and capital are computed to clear factor markets at the national level each year. Also determined endogenously are the interest and exchange rates to clear the foreign exchange and financial markets each period.

### Nesting structure

Households and firms are represented using nested CES share functions. The household nest sequence is as follows:

Savings			
Leisure			
Consumption	ENERGY & TRANSPORT	UTILITIES	Electricity Other Utilities Gas Pipeline Services
		FUELS	Natural Gas Coal Gasoline Petrochemicals
		TRANSPORT	Oil Pipeline Services Air, Rail & Bus Trucking & Storage
	GOODS	BASIC GOODS	Conventional Crude Oil Sands Agriculture Forest Products Mining
		PRODUCED GOODS	Cement Semi-durables Auto Parts & Assembly Other Manufacturing Food
	SERVICES	PROFESSIONAL SERVICES	Entertainment Construction Media, Finance Etc Sales & Retail
		OTHER SERVICES	Oil & Gas Support Education & Health

The nesting structure for firms is essentially the same except the top level combines intermediate inputs with labour and capital demand to yield output. The return to capital is the surplus after labour and inputs are paid.

Carbon dioxide emissions are tracked by fuel type. Methane emissions are determined by applying emission intensity coefficient estimates to sector output rates. More details are provided below.

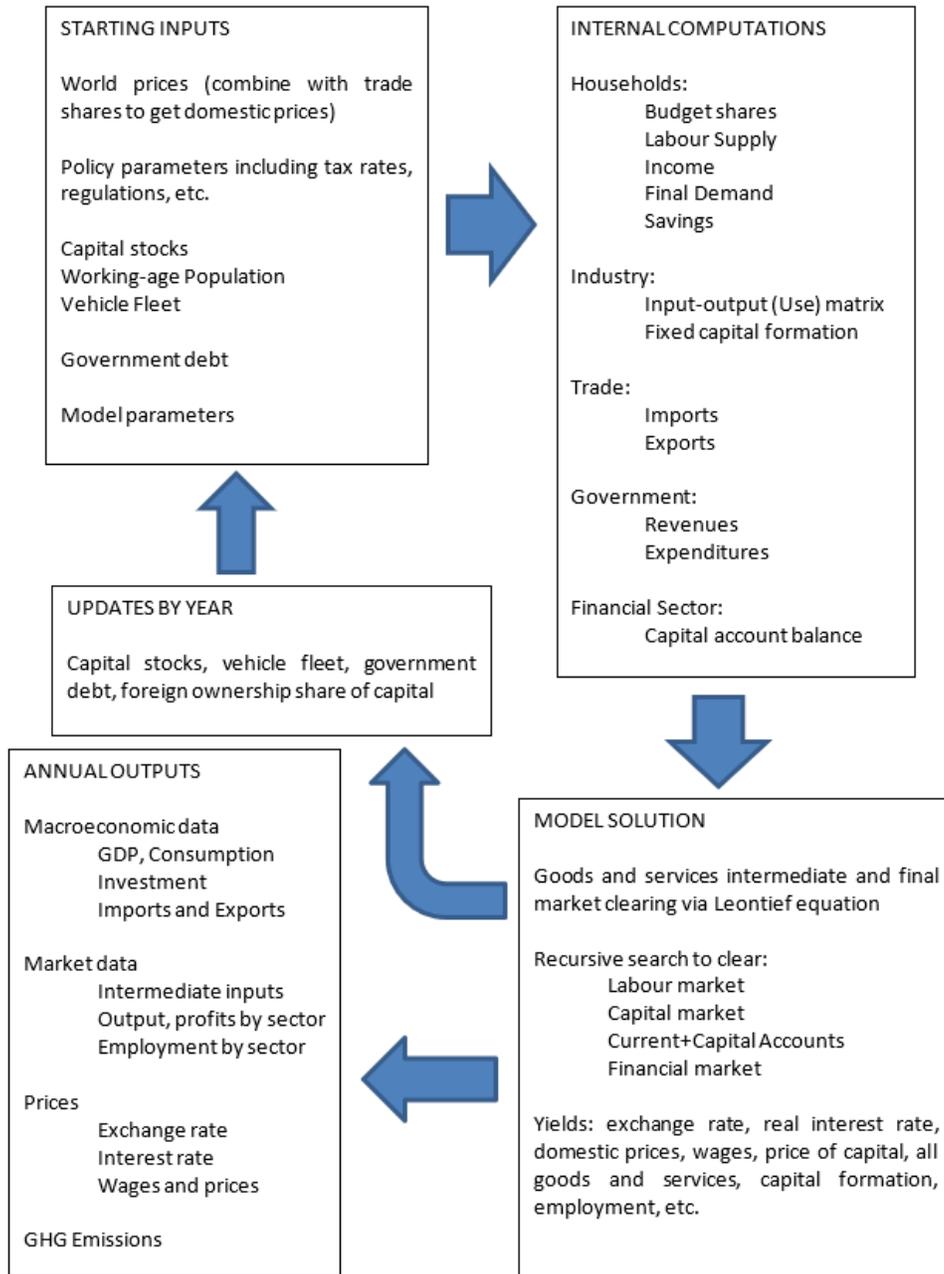
LFXCM can accommodate a unique elasticity value for each nest for each province. Initial values have been selected based on literature search and trial-and-error, but are subject to adjustment as

more information is acquired and to ensure model stability. CES function scaling parameters are calibrated to reproduce budget shares based on the 2019 StatsCan provincial IO tables.

All program components and functions are written in R.

## **2 SCHEMATIC OF MODEL COMPUTATION SEQUENCE**

The main computational steps are as described in the Figure below.



### 3 PARAMETER CALIBRATION

The LFXCM software assimilates the 2014-2018 Statistics Canada Input-Output tables each of which annually resolve 553 industries or sectors and 540 commodities in every province and territory. These are aggregated into the 26 categories listed above for 10 provinces and the far

north. The condensed tables are then used to calibrate all share parameters and tax parameters. Industrial IO and household budget shares are calibrated using the 2018 IO tables while other macroeconomic variables such as employment, greenhouse gas emissions and government budget data are set using 2019 observations.

### Factors of Production

Factors of production include labour  $L$  (by sector and province) and capital  $K$ . The capital stock is assumed to be owned jointly by households and foreigners. The user cost of capital  $q$  is computed as the price of capital less its discounted resale value net of depreciation:

$$q = p_k - p_k \frac{1 - \delta}{1 + r_t} = p_k \frac{(r_t + \delta)}{1 + r_t}$$

where  $p_k$  is the price of capital,  $r_t$  is the real interest rate and  $\delta$  is the depreciation rate. Note the user cost of capital is increasing in the real interest rate and the price of capital.

A sector's operating surplus  $\pi$  should tend in equilibrium to cover its user cost of capital, thus  $\pi = qK$ . We compute the average gross operating surplus  $\bar{\pi}$  for each sector in each province over 2014-2018 and generate the 2019 starting capital stock estimate using  $\bar{\pi}/q$ . Capital demand is determined each period as part of the computation of equilibrium and the price of capital adjusts to clear the national capital market. The capital stock then evolves using an investment function described below.

### Tax Detail

Separate intermediate tax rates by industry and province are computed using the Input-Output table values of output and input taxes net of subsidies on outputs and inputs, with the federal carbon tax added in the policy base case. Households also pay consumption taxes computed at the provincial level to take into account PST and HST rates across the province as well as the federal carbon tax levy. Households also pay income taxes which are computed using the national total income tax revenues for 2019 as recorded by Statistics Canada. The same average income tax rate applies equally to labour and capital income.

## 4 KEY MODEL FUNCTIONS

### Share Functions

These assume underlying objective functions of the CES form and are drawn from Shoven and Whalley *Applying General Equilibrium* and Berck and Sydsaeter *Economists' Mathematical Manual*.

Given a set of intermediate input prices the model determines input-output coefficients for each sector in each province. The input-output coefficients vary as relative prices change. The IO coefficients begin with the assumption that, within a nest consisting of (for example) two inputs  $(x_1, x_2)$  the firm chooses them to maximize (Berck & Sydsaeter p. 126)

$$y = \left( (a_1 x_1)^{\frac{\sigma-1}{\sigma}} + (a_2 x_2)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \text{ subject to } p_1 x_1 + p_2 x_2 = C$$

where  $\sigma =$  the elasticity of substitution. Note  $a_i = w_i^{\frac{1}{\sigma-1}}$  where  $w_i$  are the base case real shares (= nominal shares assuming base case prices = 1).

The input-output coefficients consistent with the optimal solution are

$$\frac{x_i}{y} = p_i^{-\sigma} a_i^{\sigma-1} \left( \left( \frac{p_1}{a_1} \right)^{1-\sigma} + \left( \frac{p_2}{a_2} \right)^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}}$$

The zero-profit condition implies  $p_y y = p_1 x_1 + p_2 x_2$  therefore the nest price is

$$p_y = p_1 \frac{x_1}{y} + p_2 \frac{x_2}{y}$$

The household model uses nominal shares. Given prices and total income, the consumer maximizes a utility function. Standard CES forms are:

Shoven-Whalley form:  $U = \left( \sum_i \alpha_i^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$  where  $\alpha_i$  equate to a constant multiple times base case budget shares  $w_i$ .

The optimal nominal shares according to Shoven-Whalley are:

$$x_i = \frac{\alpha_i I}{p_i^{\sigma} \sum_j \alpha_j p_j^{1-\sigma}}$$

$$\Rightarrow \frac{p_i x_i}{I} = \frac{\alpha_i p_i}{p_i^{\sigma} \sum_j \alpha_j p_j^{1-\sigma}} = \frac{\alpha_i p_i^{1-\sigma}}{\sum_j \alpha_j p_j^{1-\sigma}}$$

For a given base price vector  $p_i = 1$  we have  $w_i = \frac{x_i}{I} = \frac{\alpha_i}{\sum_j \alpha_j}$ , hence if  $\sum_j \alpha_j =$  some constant  $k$  we have  $\alpha_j = k w_j$ . Then utility can be written

$$U = \left( \sum_i (k w_i)^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

and

$$x_i = \frac{k w_i I}{p_i^{\sigma} \sum_j k w_j p_j^{1-\sigma}} = \frac{\alpha_i I}{p_i^{\sigma} \sum_j \alpha_j p_j^{1-\sigma}}$$

as before. Hence the input nominal budget shares can be used as the  $\alpha_i$  parameters for the purpose of computing revised budget shares when the prices change.

The expenditure function has the form  $E(p, y) = y(\sum_j \alpha_j p_j^{1-\sigma})^{1-\sigma}$  which means the true cost-of-living index is  $c = (\sum_j \alpha_j p_j^{1-\sigma})^{1-\sigma}$ .

### Consumer Model – Top Level

The utility function combines demand for leisure  $H$  and consumption  $C$  with associated prices  $w$  and  $p$ , time endowment  $T$  (which equals leisure  $H$  plus labour  $L$ ) and exogenous income  $Y$ . The utility function is

$$U = \frac{\gamma}{\alpha} H^\alpha + C$$

where  $\gamma$  is a scaling parameter. This is optimized against the budget constraint  $wH + pC = Tw + Y$  using a Lagrangian function

$$\ell = U - \lambda(wH + pC - Tw - Y)$$

The first-order conditions are:

$$\ell_C = 1 - \lambda p = 0 \Rightarrow \lambda = \frac{1}{p}$$

$$\ell_H = \gamma H^{\alpha-1} - \lambda w = 0 \Rightarrow H = \left(\frac{1}{\gamma}\right)^{\frac{1}{\alpha-1}} \left(\frac{w}{p}\right)^{\frac{1}{\alpha-1}}$$

These can be solved to yield a labour supply function

$$L = T - \theta \left(\frac{w}{p}\right)^\sigma$$

where  $\theta = \gamma^{\frac{1}{1-\alpha}}$  is a scaling parameter and  $\sigma = \frac{1}{\alpha-1}$  is the elasticity of leisure demand with respect to the real wage rate. The labour supply elasticity  $\epsilon_L$  is not imposed directly but is an emergent quantity based on prices, quantities and chosen parameter values. It is computed as

$$\epsilon_L = \frac{\partial L}{\partial w/p} \times \frac{\left(\frac{w}{p}\right)}{L} = -\theta \sigma \left(\frac{w}{p}\right)^\sigma / L$$

This can vary but in equilibrium is typically observed to be about 0.6.

## 5 REGULATORY RENTS AND POLICY EXPERIMENTS

The cost of certain regulations is akin to a tax-induced “Harberger triangle” or deadweight loss, except that the revenue portion does not accrue to the government instead it is dissipated and is unavailable to the economy. For example, suppose a regulation is introduced requiring construction firms to change procedures in such a way that the cost of building a home rises by 20 percent, but at the end of the process, the extra cost does not yield a 20 percent bigger house, instead the same size house has been created. In this case the production cost is scaled up by 20 percent but the increased selling price does not accrue as revenue to the home builder, instead it is offset by decreased productivity of the inputs. The LFXCM builds a number of such regulatory inefficiencies into the base case of the model, including in the electricity and refining sectors, based on relative changes over time among provinces in the marginal cost of producing equivalent outputs. The LFXCM then tracks the national costs of compliance with these regulations. No attempt is made within the LFXCM to quantify the intended benefits associated with these regulations, although such estimates can be made using the model outputs.

Policy experiments can be run in the LFXCM in which a new policy is represented in the form of changes to the pre-existing regulatory constraint structure, changes to factor supplies, changes in any of the provincial or federal tax and subsidy rates, and so forth. Numerous metrics are available for determining the costs and benefits of the policy, including provincial utility, real GDP, real consumption, employment, changes in the equity value of the capital stock, etc.

## 6 HOUSEHOLD INCOME, CONSUMPTION AND SAVINGS RATE

Household savings is assumed to be a fraction of total income determined empirically by a regression of the household savings rate on the estimated real interest rate. The data sample runs from 1991 to 2019 and the fit of the regression is highly significant. The savings rate (in % points) is given by  $HSR = 1.503 + 1.075 \times r$  where  $r$  is the nominal bank rate deflated by the consumer CPI.

In each province household incomes are derived as the sum of wages earned from employment, the domestic share of the gross operating surplus of industry and transfers from government. The amount of labour a household supplies is increasing in the real wage rate. The domestic share of ownership of capital is taken from the national balance sheet accounts which, for 2019 is set at 0.75. The domestic ownership share adjusts dynamically thereafter based on inflows of foreign capital to finance investment.

Household income minus the savings fraction yields the amount available for spending on goods and services. Since households receive income from firms in the form of both wages and capital earnings, a policy change can have multiple channels of impact. Suppose, for instance, an ethanol mandate requires refineries to use more of the output of the ethanol sector than they would have chosen to use based on market conditions. This can lead to an increase in earnings in the ethanol sector due to the increased demand, but decreased earnings due to reduced profits and reduced labour demand in the refining sector, as well as reductions in labour demand and capital earnings in other sectors if the price of fuel rises.

For this reason the ultimate effect in the model of large policy changes may be difficult to predict because they involve multiple offsetting effects. Consider the case of increased transfers to

households financed by new taxes. Increased taxes are ultimately paid by households, either because they are levied on households directly or they are levied on businesses who then pass them on to households in the form of lower wages, lower capital earnings and/or higher output prices. The extra government revenue supports new spending which increases labour demand and boosts the revenue of businesses. The overall balance of the effect in any specific province may thus be positive or negative.

## 7 MODEL SOLUTION

### Intermediate Prices

Domestic prices are a weighted average of the internal price (which for all commodities is assumed to be unity) plus the world price times the exchange rate, with the weights determined by the relative share of imports for each good or service. The world price is also assumed to be unity unless an exogenous adjustment is imposed as part of a simulation experiment (for example an increase in the world oil price).

The model adds regulatory costs and intermediate taxes (including carbon taxes) to all prices at the intermediate buyer stage. These prices are then used by all sectors in the nested CES optimization process described above to generate an endogenous input-output coefficient matrix  $A$  and unit costs of outputs.

Households take prices, government policy parameters (including transfers) and the wage rate as given. They determine the labour supply, savings and final demands based on utility maximization.

### Domestic and Foreign Trade

Initial domestic export and import levels for each province are taken from the 2019 provincial IO tables which balance to zero at the national level. The relative proportions are assumed to remain fixed but they grow over time in proportion to the growth of the labour force.

International exports and imports are calibrated using the 2019 provincial IO tables. Real export demands are adjusted using econometrically-estimated functions that estimate annual changes in export volumes by province and commodity as functions of the exchange rate, the US GDP growth rate and world prices of certain key commodities including oil and gas.

Nominal imports are adjusted using the following function:  $M^{i,j} = M_{19}^{i,j} \times p_{b,i}^{\mu} \times XR^{\sigma_x}$  where  $M_{19}^{i,j}$  is the nominal spending on imports of good  $i$  in province  $j$  in 2019,  $p_{b,i}$  is the domestic buyer price for good  $i$ ,  $\mu$  is the price response elasticity coefficient which is set to 0.1 for all goods in all provinces,  $XR$  is the exchange rate (representing the cost in Canadian dollars of purchasing a real unit of foreign goods) and  $\sigma_x$  is the elasticity of import demand with respect to changes in the exchange rate, which is set to -0.2 for all goods in all provinces. Note that appreciation of the domestic currency means  $XR$  declines.

### Government Sector

Government revenue is determined endogenously based on tax rates as described above and other policy measures as described elsewhere. Transfers to households and labour demand are fixed at

2019 levels in the policy base case and may be adjusted under policy experiments. Government purchases of goods and services are based on 2019 levels and may be adjusted in policy experiments or in growth scenarios. The government budget surplus or deficit is thus endogenous.

### Gross Fixed Capital Formation

GFCF is determined based on the assumption that average past investment spending reflected expected long run optimal levels, and future investment is adjusted using growth of the labour force and changes in the price of capital. The specific formula is

$$GFCF(j, t) = GFCF_{j,0} \times \lg(j, t) \times p_K(t)$$

where  $GFCF(j, t)$  is nominal gross fixed capital formation in year  $(t)$  in province  $j$ ,  $GFCF_{j,0}$  is the same in the calibration base year (2018),  $\lg(j, t)$  is the growth factor in the labour force in province  $j$  between 2019 and year  $(t)$  and  $p_K(t)$  is the national market-clearing price of capital in year  $t$ . Spending across categories is allocated based on 2018 budget shares.

### Net Savings and the External Sector

Household net savings plus government surplus plus investment inflows yields the funds available for investment. The latter is determined by the following equation:

$$f.funds = f_0 + f_1 \times r_t \times \chi_t^{-1}$$

where  $f_0$  and  $f_1$  are parameters estimated against recent investment flows data,  $r_t$  is the domestic real interest rate and  $\chi_t$  is the ratio of the current and previous exchange rate (noting that an increase represents a depreciation).

Total nominal GFCF determines investment needs. The real interest rate adjusts to equilibrate demand and supply of investment funds.

Currency inflows are determined by nominal exports and foreign investment funds. Currency outflows are determined by nominal imports plus payments to foreign owners of capital plus payments to foreign holders of government debt. The exchange rate adjusts to settle the current and capital accounts.

## 8 EQUILIBRIUM COMPUTATION

Within a province, given prices, tax rates, government spending and trade parameters the model yields the input-output coefficient matrix  $A$ , and final demands for consumption  $C$ , government purchases  $G$ , investment or Gross Fixed Capital Formation  $I$ , exports  $X$  and imports  $M$ . Denote  $C + I + G + X - M = F$ . If real output is denoted  $Q$  the Leontief market clearing condition is  $AQ + F = Q$ . The model solves for  $Q$  using the matrix equation

$$Q = (I - A)^{-1}F.$$

Then input-output coefficients for labour and capital are used to determine labour and capital demands by sector and province. Exogenous restrictions are imposed on the Education and Health care sector in some provinces to limit its expansion since it is primarily governed by government policy and cannot respond freely to market conditions.

Since the Leontief equation is solved for each province, and some provinces are net importers of some goods (for example, Ontario imports crude oil for refining), the equilibrium output level can be negative. If the labour IO coefficient were applied it would yield a negative demand for labour. This is also an implication of the “cross-hauling” phenomenon in which provinces can both import and export the same commodity, such as food for instance. The relevant labour demand level is therefore based on final demand before subtracting imports, which equals  $(C + I + G + X)$ . This yields, for example, an employment level of zero for oil sands production in Ontario, which is the appropriate estimate. The model uses the pre-import final demand amount as the basis for estimating labour demand in each sector and province.

The model adjusts the national wage rate to clear the national labour market, and the international exchange rate to balance the currency inflows and outflows. Provincial labour markets do not necessarily clear: there can be surpluses or shortage of labour within a province but they add up to zero nationally. The program verifies that unit profits are zero within each sector and Walras’ Law holds nationally at every iteration.

## 9 GREENHOUSE GASES AND CARBON TAXES

Greenhouse gas emissions (CO<sub>2</sub> and methane) are computed using coefficients calibrated on consumption of coal, natural gas, refined fuels and production of cement, agriculture, gas and oil operations so as to reproduce the 2019 national emissions inventory. Note that the Federal emissions inventory reports emissions by province and sector but not by fuel type. The LFXCM generates internal CO<sub>2</sub> emission coefficients by fuel type based on standard methodology that reconciles to the national emission levels, but which may not match to provincial subtotals because of the methodological differences.

### Carbon Dioxide

Sources:

Marland, G., and R.M. Rotty. 1984. Carbon dioxide emissions from fossil fuels: A procedure for estimation and results for 1950-82. *Tellus* 36(B):232-61.

BP Annual Statistical Review of Energy <https://www.bp.com/en/global/corporate/energy-economics/statistical-review-of-world-energy.html>

NRCan Energy FactBook

Emission coefficients for coal, petroleum liquids and natural gas are derived as follows.

- BP reports that in 2019 Canada consumed 2,537 thousand barrels per day of oil (620.5 million barrels of oil annually), 3.82 exajoules of natural gas and 0.78 exajoules of coal.
- Marland and Rotty estimate carbon emission coefficients for natural gas as 13.7 tC /TJ; for oil 0.85 tC/tonne oil and for coal 0.75 tC/tonne coal.

For oil, 620.5 million barrels of oil at 0.136 tonnes per barrel implies 84.4 Mt oil and 71.7 MtC. Using a conversion factor of 11/3 implies 263.0MtCO<sub>2</sub>.

For natural gas 3,820,000 TJ implies 52.3 Mt Carbon and using a conversion factor of 11/3 this implies 191.9 MtCO<sub>2</sub>.

For coal 780,000 TJ converts to mass using 29.31x10<sup>9</sup> J/t (Marland and Rotty) yielding 26.6 Mt coal, 20.0 MtC and 73.2 Mt CO<sub>2</sub>.

Canada's IPCC Emission Inventory (<https://unfccc.int/documents/65715>) lists 6Mt CO<sub>2</sub> emissions associated with cement production.

These emission totals by fuel type are then used along with the real input and output indexes within the model to generate fuel-specific emission intensity parameters.

### Methane

2019 methane emissions attributable to agriculture, oil production, natural gas production and flaring and venting were obtained from the Canadian GHG Emission Inventory. These were then attributed to the appropriate sectors, with the venting and flaring totals divided among Oil Sands production and Natural Gas production according to output shares.

### Carbon Pricing and Output-Based Pricing System Rebates

The internally-generated coefficients yield CO<sub>2</sub> emissions by fuel type which are multiplied by the carbon tax rate per tonne of CO<sub>2</sub> to yield the total carbon tax payment. The total payments per summed real input usage (in internal model units) yields an implicit carbon tax rate which is applied to prices at the intermediate stage. This process is implemented iteratively and converges on stable parameters quickly.

The Output-Based Pricing System (OBPS) system is implemented following the modeling approach in McKittrick et al. (2019). The unit cost function for sector  $i$  is

$$p_i = c_y^i + \tau(e_y^i - v \times \bar{z}_i)$$

where  $c_y^i$  is marginal cost of output,  $\tau$  is the carbon tax rate on emissions  $e^i$ ,  $e_y^i$  is marginal emissions with respect to output,  $v$  is the OBPS refund rate for the sector and  $\bar{z}_i$  is a sector-specific average emissions intensity rate. For sectors deemed energy-intensive and trade exposed  $v$  is set to 0.9, otherwise it is 0. For each sector  $i$  and each province  $j$  the value of  $\bar{z}$  is computed using input-output coefficients and emission coefficients as follows:

$$\bar{z}_i = \frac{c_{ng}a_{ng}^i y_i + c_{coal}a_{coal}^i y_i + c_f a_f^i y_i}{y_i} = c_{ng}a_{ng}^i + c_{coal}a_{coal}^i + c_f a_f^i$$

where the subscript denotes the fuel type. The OBPS subsidy for sector  $i$  is then  $\tau v \bar{z}_i$ .

Households are assumed to receive a lump-sum rebate equal to 90 percent of the total carbon tax paid on household final demand plus imports (domestic and foreign). The remaining 10 percent is added to the government budget for purchases of goods and services.

## 10 ETHANOL, CARBON CAPTURE AND ENERGY EFFICIENCY MANDATES

### Ethanol blending

The factors that affect the consumer cost of increased blending of ethanol and biofuels are chiefly the ratio of the per-litre cost of ethanol versus gasoline, the relative energy content of ethanol versus gasoline, and the supply elasticity of ethanol versus gasoline.

In the base case, Canadians are assumed to use a blend in which the fuel fraction ( $\theta_f$ ) is 95 percent and the ethanol fraction ( $1 - \theta_f$ ) is 5 percent. The per-litre blend cost is

$$P_b = \theta_f P_f + (1 - \theta_f) P_e$$

where  $P_b$  is the blend,  $P_f$  is the gasoline price and  $P_e$  is the ethanol price. We assume that the  $P_f$  is fixed by the world supply price, but the price of ethanol follows an upward-sloping supply curve with an elasticity of  $\sigma = 0.237$  based on [Luchansky and Monks \(2009\)](#). To account for the relative size of the Canadian and US markets this parameter value is reduced by 80 percent to 0.047. Finally denote the ratio of the price of fuel to that of ethanol by  $R_f = P_f/P_e$ .

If the fuel blending requirement changes to  $\theta'_f$  the percent change in the ethanol fraction compared to the base case is  $(0.95 - \theta'_f)/0.05$ . A one percent change in the blend requirement may represent substantially more than a one percent increase in the Canadian supply requirement but we will assume the percent change in required supply corresponds to the percent change in the blend requirement. The new cost of ethanol production as a result of a new content requirement is therefore

$$P_e = 0.9 \times \left( 1 + \sigma \frac{(0.95 - \theta'_f)}{0.05} \right).$$

The price adjustment factor resulting from the new blending requirement can therefore be written

$$A_p = \frac{\left( \theta'_f \times R_f + (1 - \theta'_f) \times \left( 1 + \sigma \frac{(0.95 - \theta'_f)}{0.05} \right) \right)}{\theta_f R_f + (1 - \theta_f)}.$$

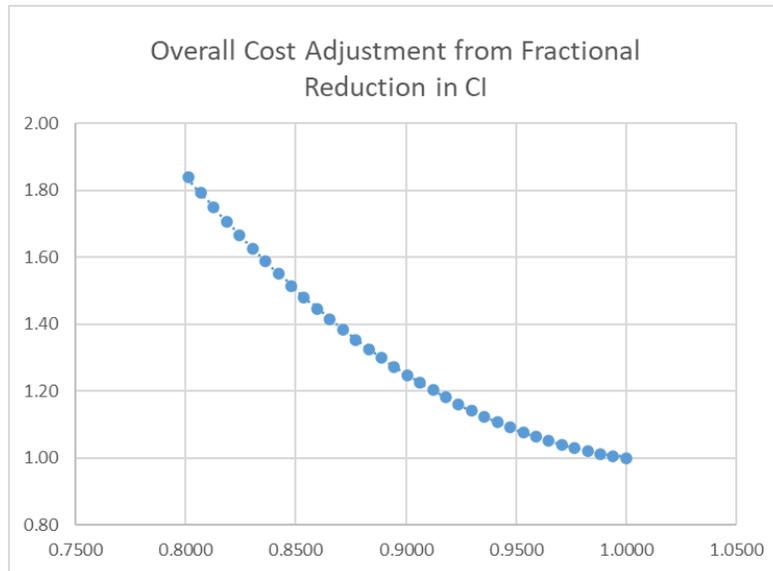
Ethanol contains only 67 percent of the energy in petroleum fuel. Therefore the energy output of the blend is  $E_b = (\theta'_f + 0.67 \times (1 - \theta'_f))$ . In the base case  $E_b = 0.95 + 0.67 \times 0.05 = 0.9835$ . Therefore the adjustment factor for the energy output of the mandated blend will be

$$A_b = \frac{E_b}{0.9835} = \frac{(\theta'_f + 0.67 \times (1 - \theta'_f))}{0.9835}.$$

The combined adjustment factor for the cost of fuel will therefore be

$$AF = \frac{A_p}{A_b}$$

Based on the text of the 2022 CFS regulation we assume gasoline has a baseline carbon intensity (CI) of 95.0 gCO<sub>2</sub>e/MJ, while that of ethanol is assumed to be 41.0 (Hosseini et al. 2019<sup>1</sup>). The above formulas were used in a spreadsheet model with varying values of  $\theta'_f$  (ranging from 0.61 to 0.95) and an assumed value of  $R_f = 1$ , implying cost parity between ethanol and gasoline on a per litre basis, which yielded the following schedule between the fraction of allowed CI relative to the base case on the horizontal axis and the relative cost to the consumer of an energy-equivalent volume of fuel on the vertical axis. For example, a 10 percent reduction in CI (corresponding to the point 0.9 on the horizontal axis) implies a cost adjustment factor of 1.25, or a 25% increase in fuel costs. Of this, it can be shown that about 18 percent of the increase is due to the increase in the price of ethanol itself due to the supply elasticity and the remainder is the cost due to the lower energy content of the fuel blend.



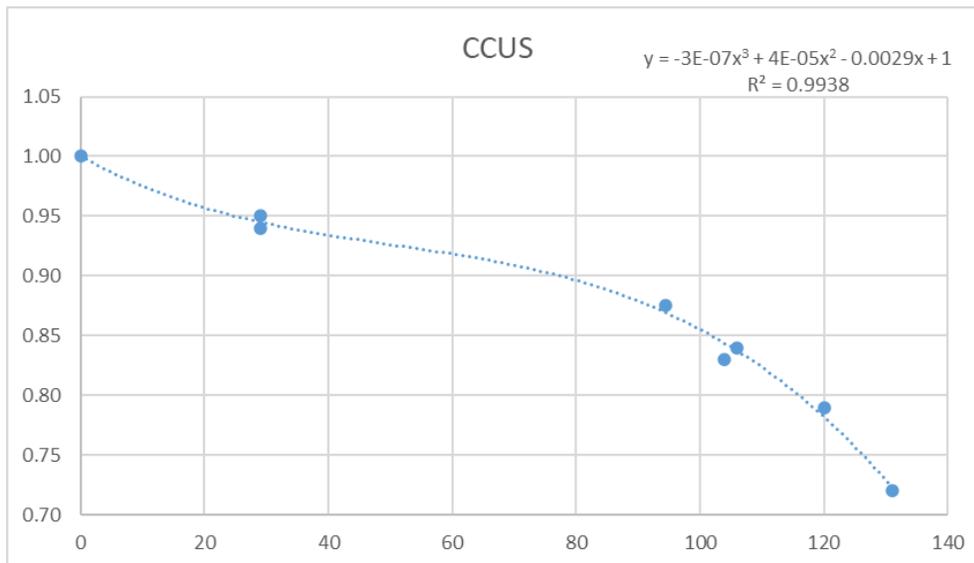
The schedule shown was fitted to a second-degree polynomial for application in the LFX model. The entire schedule varies somewhat as the price ratio  $R_f$  varies, and it was found that the result could be closely approximated by multiplying the polynomial by the inverse of  $R_f$ , in other words if ethanol gets more expensive relative to gasoline the schedule rotates clockwise by the same proportion.

<sup>1</sup> Hosseini, H, Millington, D, Romaniuk, A. (2019). Economic and Emissions Impacts of Fuel Decarbonization. Canadian Energy Research Institute.

The Adjustment Factor schedule is then used to guide the forced spending on alternative fuels. It is assumed that 90 percent of the mandated increase in gasoline costs translates into demand for ethanol, of which some is imported and some is produced domestically depending on the province.

### Carbon Capture, Utilization and Storage

In the base case of the LFXCM5 model the carbon price creates a small incentive for CCUS but only on a limited basis, according to a schedule which implies a \$50 per tonne tax leads to sequestering of 1.4 percent of CO<sub>2</sub> emissions from all intermediate use of coal, natural gas and liquid fuels across all industrial sectors. The proposed ERP tax credit as of 2025 combined with assumed provincial and federal incentives leads to accelerated adoption of CCUS. The model component that implements this is based on the survey of levelized cost estimates in [Irlam \(2017\)](#), which pairs emission reduction rates with associated costs per tonne of avoided emissions. Converting the US\$ per-tonne cost rates into inflation-adjusted Canadian dollars and adjusting for the investment tax credit leads to the cost schedule shown in Figure 4.2.



CCUS Cost Schedule. Data adapted from Irlam (2017). Horizontal axis: carbon price (CDN\$). Vertical axis: fraction of oil and gas sector emissions sequestered.

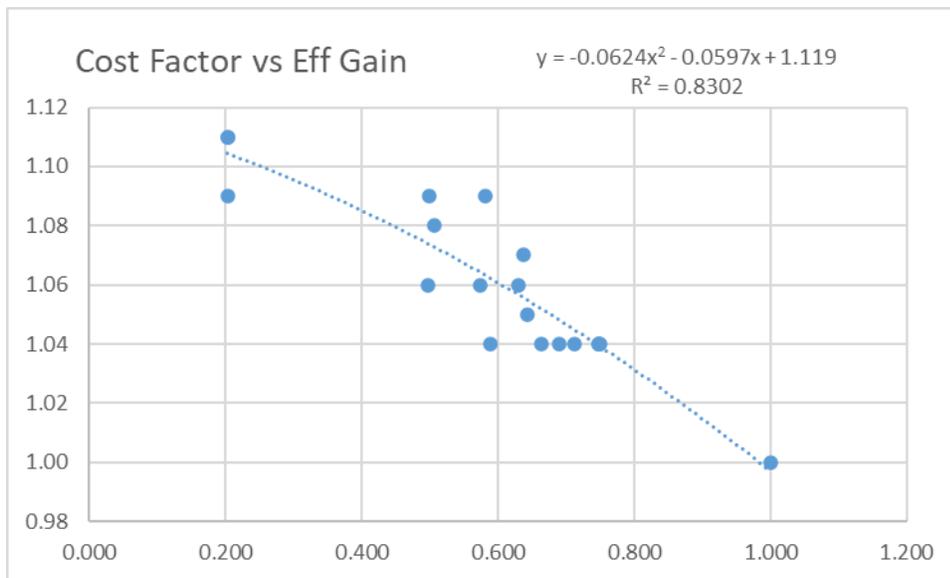
The Figure shows, on the vertical axis, the emission adjustment factor and, on the horizontal axis, the carbon price net of the tax credit. A 3<sup>rd</sup>-order polynomial curve is fitted through the data and yields the function shown in the top right corner. Because the curve accelerates downward above \$100 per tonne, whereas the principle of diminishing returns would suggest it should level out, this curve maybe over-optimistic in how much CCUS accomplishes at higher carbon prices, but it is nonetheless used as is.

Substituting the carbon tax value in for  $x$  yields the emission factor  $y$ . A tax of, for example, \$120.15 per tonne yields a predicted emissions factor of 0.78, which means CCUS adoption induced by the carbon price reduces the emissions of intermediate fossil fuel use in Canadian industry by 22

percent. Using this function, the model assumes that at a carbon tax of about \$195 per tonne all intermediate industrial fossil fuel-related CO<sub>2</sub> emissions would be captured and stored.

### Energy Efficiency Mandates

EE mandates for buildings and building components have compounding effects. Some characteristic values relating required targets for energy use (heating and cooling) and construction costs was set out for standard residential buildings in CHBA (2018). When graphed and tabulated they can be summarized in the Figure below. The horizontal axis shows the building efficiency improvement requirement as a proportion of the base case (for instance, a required 10 percent efficiency gain corresponds to a value of 0.9). The vertical axis shows the effect on the price of construction as a multiple of the base case value. A second-order polynomial line of best fit is shown. Note that these cost estimates do not include a profit margin for builders. Also the concavity of the line suggests diminishing marginal costs which is implausible given diminishing returns to investments in energy efficiency. Nonetheless the fitted curve was used as it to assign a construction cost adjustment factor to prescribed efficiency improvements.



Construction Cost Price Index versus Efficiency Gain Requirements.

## 11 MOTOR VEHICLE STOCKS AND FLOWS

The LFX model takes passenger vehicle data from the [NRCan](#) Comprehensive Energy Use Database. For each province the latest (2018) data on the stock of cars per province, sales of new cars and fraction of cars that are electric are used to initialize the stock-flow model. Assuming a 3% scrappage rate by province the 2019 stocks are estimated by computing 2018 sales as a fraction of the vehicle stock and using that as the 2019 value. It is assumed that EVs have only had substantial market presence for the past 5 years, and on that basis the fraction of annual sales assigned to

electric vehicles is taken to be 3 times the current stock fraction (a more precise fraction can be derived analytically but in the steady-state the 3x approximation works well). For those provinces currently showing 0% EVs (SK, MA, NB, NS, PEI and NF) the sales fraction is set to 0.3%. EV sales are constrained to be 0 in the Far North.

Base case vehicle fleet characteristics are determined as follows. The number of years between 2019 and the simulation year is denoted  $d.y$ . An iterative loop is used for each province which generates updated vehicle stock and EV stock estimates based on an assumed 3% scrappage rate and the 2019 sales-to-stock ratio. Then for each province  $j$  the current EV fraction, denoted  $f_j^{EV}$  is determined. This is used to adjust the fuel demands of households by multiplying the basic amount by  $(1 - 0.7 \times f_j^{EV})$ . The 0.7 adjustment parameter takes into account a 30% rebound effect by which reduced demand for fuels on the part of EV owners lowers the cost of fuels for drivers of conventional cars, prompting them to purchase more. Household electricity demand is also modified using an adjustment factor  $(1 + 0.4 \times f_j^{EV})$  where the 0.4 parameter denotes the estimate that if, for example, 10 percent of the vehicle fleet in a province were to become electric, total household electricity consumption would rise by 4 percent.

In a policy experiment case in which regulations force EV sales to become a larger fraction of new vehicle sales, the cost of the regulation is represented by a regulatory rent parameter increase on automobile sales. Fuel efficiency regulations that raise the cost of new car purchases are known to slow down the replacement of used cars (see [Dou and Linn 2020](#)) so this is captured by reducing the new sales rate by the factor  $(\% \text{ increase in purchase cost}) \times 0.3$  where 0.3 is the elasticity parameter estimated in [Leard 2022](#).

## 12 OUTPUTS

For a policy experiment the model is run twice, under the base case and policy experiment assumptions. Changes in all of these outputs are estimated (in most cases by sector and province):

- Nominal and Real GDP by Expenditure
- Real GDP Per Worker
- Nominal and Real Household Consumption
- Employment (Labour Supply, Labour Demand and Market Surplus)
- Real Exports
- Real Imports
- Investment spending
- Household savings
- Cost of Living Index
- Capital Returns Relative to Average
- Government Revenue (Labour and Capital Income Taxes, Indirect Taxes)
- Government Spending including Interest on Debt
- Government Budget Surplus
- Income Tax Rate and Overall Tax Shares
- Current and Capital Account Balances, Exchange Rate Index
- CO2 and Methane Emissions by Source
- Purchase Prices faced by Households

- Capital-labour Ratios

## **13 REVISIONS SINCE PREVIOUS VERSION**

Corrections and revisions compared to the most recent version of the LFXCM include the following:

- A correction was made to the formula for summing indirect tax payments by industry at the intermediate stage which was incorrect in previous versions
- Capital income (net operating surplus) had previously been assumed to be entirely paid to domestic households; it is now divided up between domestic and foreign owners according to observed ownership shares
- The OBPS subsidy was parameterized previously in a way that overstated its magnitude
- Taxes and regulatory measures were allowed to propagate once through the intermediate economy prior to the final demand sector responding, implying different agents faced different price vectors, but this ad hoc mechanism has been removed and all agents face consistent prices
- The household savings rate is now determined as a function of the interest rate rather than being fixed
- The motor vehicle stock-flow model with fuel use and electricity demand adjustments was added
- All base-case input-output coefficients are computed using share functions based on 2018 nominal input-output table values.